

$$1(a) 12x + 6 = 18 - 24$$

$$12x = -6 - 6$$

$$12x = -12$$

$$x = -1$$

$$(b) -2x^2 + 2 = -3x$$

$$-2x^2 + 3x + 2 = 0$$

$$-(2x-1)(x+2) = 0$$

$$2x-1=0 \quad \text{or} \quad x+2=0$$

$$2x=1$$

$$x=-2$$

$$x = \frac{1}{2}$$

$$x = \frac{1}{2}, -2$$

$$2. \quad y = x^2 - 4$$

$$\frac{5-y}{3} = x$$

$$5-y = 3x$$

$$y = 5-3x$$

$$5-3x = x^2 - 4$$

$$x^2 + 3x - 9 = 0$$

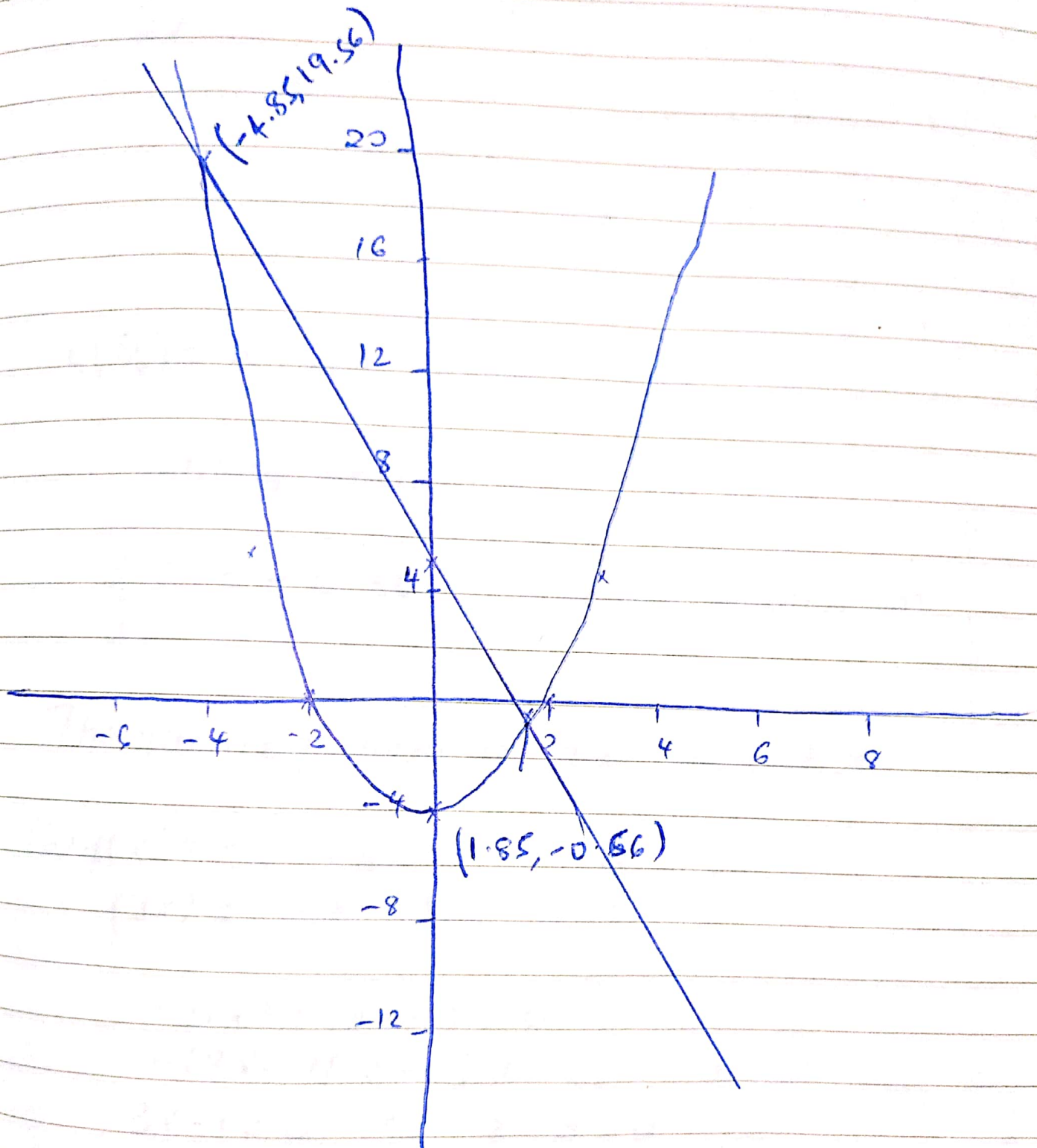
$$x = \frac{-3 \pm \sqrt{9+36}}{2} = \frac{-3 \pm 3\sqrt{5}}{2}$$

$$\text{For } x = \frac{-3+3\sqrt{5}}{2}, \quad y = 5 - 3\left(\frac{-3+3\sqrt{5}}{2}\right)$$

$$= \frac{+19-9\sqrt{5}}{2}$$

$$\text{For } x = \frac{-3-3\sqrt{5}}{2}, \quad y = 5 - 3\left(\frac{-3-3\sqrt{5}}{2}\right)$$

$$y = \frac{19 + 9\sqrt{5}}{3}$$



$y = x^2 - 4$ is a parabola opening upwards.

$\frac{5-y}{3} = x$ is a straight line trending downwards from left to right.

3 (a) Mean.

$$\text{Mean (\% managers)} = \frac{23 + 28 + 28 + 40 + 40 + 42 + 44}{7} \\ = 35\%$$

$$\text{Median (\% managers)} = 40\%$$

Median is the appropriate measure of central tendency

$$\text{Mean (profits)} = \frac{16 + 14 + 22 + 24 + 10 + 28 + 12}{7} \\ = £18 \text{ m}$$

$$\text{Median (profits)} = £16 \text{ m}$$

Mean is the appropriate measure of central tendency.

(b) Standard deviation.

X	$X - \bar{x}$	$(X - \bar{x})^2$
23	-12	144
28	-7	49
28	-7	49
40	5	25
40	5	25
42	7	49
44	9	81
Σ		442

$$s = \sqrt{\frac{442}{7-1}} = 8.387$$

The data points are 8.387 units away from the mean

X	$X - \bar{X}$	$(X - \bar{X})^2$
16	-2	4
14	-4	16
22	4	16
24	6	36
10	-8	64
28	10	100
22	-6	36
Σ		272

$$S = \sqrt{\frac{272}{7-1}} = 6.733$$

The data points are 6.733 units away from the mean.

(C)	X	Y	XY	X^2	Y^2
	23	16	368	529	256
	28	14	392	784	196
	28	22	616	784	484
	40	24	960	1600	576
	40	10	400	1600	100
	42	28	1176	1764	784
	44	12	528	1936	144
Σ	245	126	4440	8997	2540

$$r = \frac{n \Sigma XY - \Sigma X \cdot \Sigma Y}{\left((n \cdot \Sigma X^2 - (\Sigma X)^2) (n \Sigma Y^2 - (\Sigma Y)^2) \right)^{1/2}}$$

$$= \frac{7(4440) - (245)(126)}{\left((7(8997) - 245^2) (7(2540) - 126^2) \right)^{1/2}}$$

$$= 0.089$$

Since the Pearson correlation coefficient is close to 0, there is no relationship between the two variables.

$$4. (a) y = -3x^4 + 10x^2 + 26$$

$$\frac{dy}{dx} = -12x^3 + 20x$$

(General rule)

$$(b) p = (q^2 + 3)(2 - 3q^3)$$

$$\frac{dp}{dq} = (q^2 + 3)(-9q^2) + (2 - 3q^3)(2q)$$

$$= -9q^4 - 27q^2 + 4q - 6q^4$$

$$= -15q^4 - 27q^2 + 4q \quad (\text{product rule})$$

$$(c) p = \frac{q^2 - 2}{5 - 3q^3}$$

$$\frac{dp}{dq} = \frac{2q(5 - 3q^3) - (-9q^2)(q^2 - 2)}{(5 - 3q^3)^2}$$

$$= \frac{10q - 6q^4 + 9q^4 - 18q^2}{(5 - 3q^3)^2}$$

$$= \frac{3q^4 - 18q^2 + 10q}{(5 - 3q^3)^2} \quad (\text{Quotient rule})$$

$$(d) p = (q^2 - 2q^3)^5$$

$$\frac{dp}{dq} = 5(q^2 - 2q^3)^4(2q - 6q^2)$$

(chain rule)

$$5(a) \ y = 10x - 3x^2$$

$$\frac{dy}{dx} = 10 - 6x$$

$$10 - 6x = 0$$

$$6x = 10$$

$$x = \frac{10}{6} = \frac{5}{3}$$

$$y\left(\frac{5}{3}\right) = 10\left(\frac{5}{3}\right) - 3\left(\frac{5}{3}\right)^2 = \frac{25}{3}$$

$$\text{Turning point} = \left(\frac{5}{3}, \frac{25}{3}\right)$$

$$\frac{d^2y}{dx^2} = -6 < 0 \quad (\text{Local maximum})$$

The turning point is local maximum

$$(b) \ f(x) = -x^3 + 5x^2 - 3x - 6$$

$$f'(x) = -3x^2 + 10x - 3$$

$$-3x^2 + 10x - 3 = 0$$

$$-(3x-1)(x-3) = 0$$

$$3x-1=0 \quad \text{or} \quad x-3=0$$

$$x = \frac{1}{3} \quad \text{or} \quad x = 3$$

$$f\left(\frac{1}{3}\right) = -\left(\frac{1}{3}\right)^3 + 5\left(\frac{1}{3}\right)^2 - 3\left(\frac{1}{3}\right) - 6 = -\frac{175}{27}$$

$$f(3) = -(3)^3 + 5(3)^2 - 3(3) - 6 = 3$$

$$\text{Turning points} = \left(\frac{1}{3}, -\frac{175}{27}\right), (3, 3)$$

$$f'(x) = -6x + 10$$

$$f'(\frac{1}{3}) = -6(\frac{1}{3}) + 10 = 8 > 0 \quad \text{Minimum}$$

$$f'(3) = -6(3) + 10 = -8 < 0 \quad \text{Maximum.}$$

There is a local minimum at $x = \frac{1}{3}$ and local maximum at $x = 3$

$$6 \quad TC = 4dq - \frac{1}{2}eq^2 + f$$

$$\frac{d(TC)}{dq} = 4d - eq$$

$$4d - eq = 0$$

$$eq = 4d$$

$$q = \frac{4d}{e}$$

e should not be zero

$$TC = 4d\left(\frac{4d}{e}\right) - \frac{1}{2}e\left(\frac{4d}{e}\right)^2 + f$$

$$= \frac{16d^2}{e} - \frac{8d^2}{e} + f$$

$$7. (a) \int (6 - 3x^4 + 4x^{-2}) dx$$

$$= 6x - \frac{3}{5}x^5 - 4x^{-1} + C$$

$$= 6x - \frac{3}{5}x^5 - \frac{4}{x} + C$$

$$(b) \int \frac{1}{2x} dx = \frac{1}{2} \int \frac{1}{x} dx$$

$$= \frac{1}{2} \ln|x| + C$$

$$8. \int_1^3 (6x^2 - 3x + \frac{1}{3}) dx$$

$$\left[\frac{6}{3}x^3 - \frac{3}{2}x^2 + \frac{1}{3}x \right]_1^3$$

$$\left[2x^3 - \frac{3}{2}x^2 + \frac{1}{3}x \right]_1^3$$

$$\left(2(3)^3 - \frac{3}{2}(3)^2 + \frac{1}{3}(3) \right) - \left(2(1)^3 - \frac{3}{2}(1)^2 + \frac{1}{3}(1) \right)$$

$$= \frac{83}{2} - \frac{5}{6} = \frac{122}{3}$$

